

Generation of Intense Black-Body Radiation in a Cavity by a Laboratory Pulsed Power Source

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Z. Naturforsch. **41a**, 463–467 (1986); received November 14, 1985

Pulsed laser and particle beams afford possibilities of generating intense black-body radiation in small cavities of high-Z material. The radiation confinement in the cavity is investigated on the basis of the non-stationary hydrodynamic equations including radiative heat transfer. It is estimated that with a pulse energy of 10^{12} erg a temperature of about 5×10^6 K may be obtainable.

PACS 47.70.Mc; 44.40.+a

Pulsed laser and particle beams offer prospects of laboratory generation of black-body radiation [1] with temperatures exceeding 10^6 K. The enormous radiant energy flux of $\geq 10^{12}$ W cm $^{-2}$ which, according to the Stefan-Boltzmann law, accompanies such high-temperature radiation makes this possibility of great scientific and technical interest.

The principle is to radiate the source energy into a cavity with a solid wall. With a laser source, a small hole for transmitting the beam would be required; a particle beam could possibly penetrate a wall thin enough to transmit the beam but thick enough to contain the radiation in the cavity. The rapid deposition of energy in the cavity is supposed to heat the inner wall to a high temperature and to generate an intense black-body radiation field in equilibrium with the wall. For the applications envisaged it is important that at these elevated temperatures the radiative exchange of energy between different wall elements could become so effective that it might be possible to establish very uniform conditions in the cavity even if the initial energy deposition by the source is not uniform. Furthermore, because photons propagate with the speed of light, the radiation may be formed in the empty cavity before it fills with evaporated material from the wall.

To attain high temperatures in the cavity, it is obviously necessary to provide some confinement of the energy transferred from the source to the radiation field. The confinement of the radiation

cannot be achieved by ordinary reflection at the wall because reflection is insignificant for the soft x-rays constituting the bulk of the Planck distribution at such high temperatures. The only possibility is provided by the same mechanism responsible for radiation confinement in the sun: the presence of wall material forces the radiation to diffuse and thus reduces the outward flux of energy, providing at the same time partial re-emission of the energy back into the cavity. The major difference to the sun is that radiation with the envisaged temperature can only be maintained for a very short time in the laboratory; we are thus faced here with an extremely time-dependent situation.

The temperature of an intense radiation field in a small, closed cavity will be determined by a balance of the energy received from the source and that lost to the wall. The loss of energy is connected with the heating of new layers of material as the radiation diffuses into the wall and with the conversion of internal energy into kinetic energy as the heated material begins to expand. Radiation confinement in a cavity is therefore a hydrodynamic problem of radiative heat transfer [2–4].

In this short communication we shall estimate the maximum temperature attainable with a source of given power (a more detailed discussion of the subject being given elsewhere [5]). Our analysis is based on the following major assumptions: (i) local thermodynamic equilibrium (LTE) between radiation and matter, (ii) lossless coupling of the source to the radiation heat bath in the cavity (thus ignoring the physical nature of the source and problems connected with the thermalization of its energy), and (iii) planar geometry. The latter

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assumption is valid for a cavity where the expansion of the heated wall material remains restricted to the vicinity of the wall up to the end of the heating pulse. Furthermore, we assume that the Rosseland mean free path of the radiation l_R can be represented in the form of a power law (v is the specific volume, A_l is a constant) as

$$l_R = A_l T^j v^{\mu'} \quad (1)$$

This formulation is equivalent to a power law for the coefficient of radiative heat conduction of the form

$$\kappa = \kappa_0 T^v v^\mu,$$

where (see [2], p. 654) $\kappa_0 = (16/3) \sigma A_l$ (σ is the Stefan-Boltzmann constant) and $v = j + 3$ and $\mu' = \mu$. The hydrodynamic equations in planar geometry, including radiative conduction but not radiation pressure and energy, are [2]

$$\partial v / \partial t - \partial u / \partial m = 0,$$

$$\partial u / \partial t + \partial p / \partial m = 0,$$

$$(1/(\gamma - 1)) (\partial / \partial t) (p v) + p \partial v / \partial t \\ = a (\partial / \partial m) [(p v)^\lambda v^{\mu-1} (\partial / \partial m) (p v)].$$

Here m is the Lagrangian mass coordinate, u, v, p are, respectively, the velocity, specific volume, and pressure, and γ is the adiabatic index. To put the hydrodynamic equations into the form given above, we took the specific internal energy e as

$$e = p v / (\gamma - 1),$$

and the equation of state as

$$p v = K T^\delta.$$

The constant a is related to κ_0, K, v and δ by $a = \kappa_0 / (\delta K^{(v+1)/\delta})$. The exponent λ is related to v and δ by $\lambda = (v + 1 - \delta) / \delta$.

These equations have to be solved with the appropriate boundary condition for a cavity. It turns out [5] that for the small cavities involved and for the temperatures achievable with a laboratory source transit time effects and the storage of energy in the volume of the cavity may be neglected. For our discussion we assume further that the inner wall of the cavity is uniformly irradiated by the source. In this case the wall temperature is also uniform and the thermal radiation in the cavity isotropic. Consequently, for the closed geometry of a cavity, the

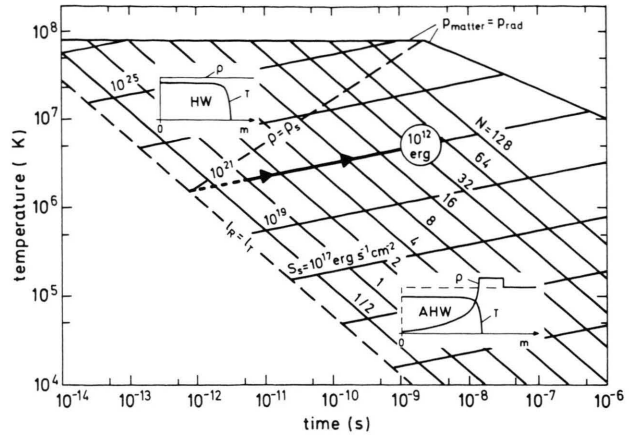


Fig. 1. Increase of the temperature and the quality factor N for radiant confinement with time in a cavity irradiated at a given source flux S_s (most favourable case based on the maximum opacity theorem). The circle marks the conditions achievable in a 2 mm radius cavity heated by a 10^{12} erg/2 ns pulse (see text).

radiant energy fluxes incident and re-emitted by the wall cancel exactly. There is, however, a net flux of heat into the wall which is supplied by the source at the vacuum/material interface. Thus, in the limit of radiation heat conduction, the appropriate boundary condition for a cavity is that the net flux from the bath to the wall be equal to the given source flux on the wall.

The problem of a heat bath in contact with a wall is in fact a classical problem of the physics of high-temperature hydrodynamic phenomena. As is well known the penetration of heat into a wall at such high temperatures occurs in the form of a non-linear heat wave [2–4] (for the profiles of the waves see insets of Figure 1). Initially, the penetration of heat occurs so rapidly that the density of the material does not change during the heating. A detailed discussion of the heat wave (HW) propagating in a medium of constant density is given in [2]. However, with time expansion of the heated material sets in and must be taken into account. The heat wave with expansion, called the ablative heat wave (AHW), was recently discussed by the present authors [4]. Both waves are self-similar and solve in principle the radiation confinement problem in the LTE approximation.

Of greatest interest for this discussion are the scaling laws for the characteristic or mean values of the hydrodynamic variables, in particular the tem-

perature, rather than their spatial profiles. These scaling laws can be obtained by dimensional analysis from the governing parameters (we follow here the notation of [6]) of the problem alone, without recourse to the self-similar solutions.

By inspecting the hydrodynamic equations and the boundary conditions the governing parameters for the two waves are found to be

$$t, S_s, a v_s^{\mu-1} \quad \text{for the heat wave (HW)}$$

and

$$t, S_s, a \quad \text{for the ablative heat wave (AHW)}.$$

This means that the problem depends on time t , the heat flux S_s into the wall given as boundary condition, and the parameters $a v_s^{\mu-1}$ (v_s is the specific volume corresponding to solid density) or a , characterizing the radiative heat conduction.

It may be worthwhile to emphasize [4] that the ablative heat wave does not depend on the solid density through a boundary condition with the undisturbed material. The reason is that this boundary condition may be simplified by the assumption of an infinitely dense solid because the characteristic density of the expanding material decreases with time and asymptotically only momentum, but no energy is transferred to the solid. It has been verified by computer simulations [7] that the approximation to the real flow is excellent as soon as the characteristic density of the wave drops below solid density.

We use the cgs system of mechanical units with three independent basic units: [length] = L , [time] = θ , and [mass] = M . The dimensions of the governing parameters are in this case $[t] = \theta$, $[S_s] = M\theta^{-3}$, $[a v_s^{\mu-1}] = M^2 \theta^{2\lambda-1} L^{-2(\lambda+2)}$, $[a] = M^{\mu+1} \theta^{2\lambda-1} \cdot L^{-(2\lambda+3\mu+1)}$. Forming by dimensional analysis a product of the governing parameters with the dimension of a specific energy ($[e] = [p v] = L^2 \theta^{-2}$) and expressing the result in terms of the temperature through $p v = K \cdot T^\delta$, one obtains

$$T \cong K^{-1/\delta} (t S_s^2 a^{-1} v_s^{1-\mu})^{1/\delta(\lambda+2)}, \quad \text{(HW)} \quad (2)$$

$$T \cong K^{-1/\delta} (t S_s^{\mu+1} a^{-1})^{2/\delta(2\lambda+3\mu+1)}. \quad \text{(AHW)} \quad (3)$$

These expressions describe the evolution of the temperature with time for a given source flux S_s . Within the frame of their derivation they are numerically accurate up to a factor of order unity which may be obtained from the self-similar solution [2–5]. For their evaluation K , a and v_s have to be specified.

The temperature in the cavity will be the higher, the larger the opacity of the wall material is. In order to obtain an upper bound on the attainable temperature, we choose for the opacity $(\varrho l_R)^{-1}$ the expression given by the Bernstein and Dyson maximum opacity theorem [8]

$$(\varrho l_R)^{-1} = (Z/A) (R_y/kT) 4.57 \times 10^5 \text{ cm}^2 \text{ g}^{-1}.$$

Here Z/A is the ratio of the nuclear charge to the atomic number of the wall material, R_y the Rydberg constant and k the Boltzmann constant. Comparison with (1) shows that in this case $(j, \mu') = (1, 1)$ and, with $Z/A \cong 0.4$ for a high- Z material, $A_l = 3.5 \times 10^{-11} \text{ g cm}^{-2} \text{ K}^{-1}$. In the equation of state we choose $\delta = 3/2$ and $K = 7.4 \times 10^3 \text{ cm}^2 \text{ s}^{-2} \text{ K}^{-3/2}$ as an approximation to the SESAME library [9] for gold (note that $\gamma \cong 1.25$ due to material ionization). One then obtains $v = j + 3 = 4$, $a = [(16/3) \sigma A_l / \delta] K^{(v+1)/\delta} = 8.8 \times 10^{-28} \text{ g}^2 \text{ cm}^{-26/3} \text{ s}^{11/3}$, and $\lambda = 7/3$. We note that for the particular choice $(j, \mu') = (1, 1)$ (not in general), (2) and (3) become identical, i.e. the onset of expansion does not affect the heat loss to the wall (within a factor of order unity). One obtains from (2) and (3) the single relation

$$T \cong 38 S_s^{4/13} t^{2/13}. \quad (4)$$

For the following discussion we define the quantity

$$N = \sigma T^4 / S_s = (\text{circulating radiant flux/source flux}). \quad (5)$$

N may be called the intensity enhancement factor or, by analogy with the Q -factor of a microwave cavity, the quality factor for radiation confinement. It is numerically equal to the number of re-emissions of the source energy in the cavity before it is lost to the wall. The dependence of N on T and t may readily be obtained by substituting the expression (4) in (5).

Figure 1 shows graphically lines of constant S_s and N in the $T-t$ plane together with border lines marking the range of validity of the solutions. For the following we shall assume that a temporally constant source flux S_s is directed against the wall; the development of the temperature with time is then obtained by following a line of constant S_s from left to right.

On the left-hand side of the border line $l_R = l_T$ (l_T is the temperature gradient length) the waves are not optically thick ($l_R > l_T$) and the LTE approximation is not valid. However, unless the source

flux S_s is very weak, this transient stage is of much shorter duration ($\cong 10^{-11}$ s) than typical pulse durations of 10^{-9} – 10^{-7} s. After the border line has been crossed equilibrium is gradually approached with time, the given scaling laws being expected to become the more accurate the deeper the flow develops into the LTE regime on the right-hand side of this border line.

The LTE regime is divided by the border line $\varrho = \varrho_s$ into (see insets) the heat wave (HW) and the ablative heat wave (AHW) region. The line $\varrho = \varrho_s$ is defined by the condition that the characteristic density of the ablative heat wave has decreased from infinity (as assumed for self-similarity) to solid density; simultaneously, on this line the condition is fulfilled that the propagation velocity of the heat wave (HW) has decreased to sound velocity. Because, as was already noted, for $(j, \mu') = (1, 1)$ the onset of expansion does not affect the heat loss to the wall, the slope of the lines of constant S_s does not change when they cross this border line.

The temperature is seen to rise with time in Figure 1. This is due to the fact that the energy loss into the wall decreases with time because the radiation must diffuse through more and more hot material. The improvement in the quality of radiation confinement is manifested in the increase of N along lines of constant S_s . If the source flux is intense enough and lasts for a sufficiently long time, N may attain values much larger than one.

As N gets large the establishment of uniform conditions in a real cavity with not perfect uniform irradiation can be expected through multiple re-emissions of the heating power. Furthermore, multiple re-emissions provide an efficient transfer of the source energy to a hole or an absorbing object in the cavity (the energy finally arrives at the hole or object even if it is small). A cavity with highly reemitting walls may be considered as a concentrating device which directs the energy at an enhanced flux level to the hole or absorbing object. This point is important if the cavity serves as a radiation source or is used to irradiate dense matter for the generation of high pressures.

The temperature in the cavity increases with S_s , the source flux on the inner wall. Thus, for a source of given power, the cavity should obviously be made as small as possible. However, even if the source radiation can be very well focussed, additional constraints will limit the minimal size of the

cavity. With a laser source, for example, the plasma density in the cavity due to filling by expanding wall material must not exceed the so-called critical density because otherwise the laser beam cannot propagate. Whatever the constraint is, let us specify the radius by the condition

$$h = R/(s\tau). \quad (6)$$

Here h is a prescribed safety factor which determines how much the radius of the cavity is larger than the distance which the heated material expands from the wall (s is the sound velocity, τ the pulse duration). For a given h the density at the centre of the cavity is of order (the ablative heat wave being approximated on its vacuum side by an isothermal rarefaction wave, see [7]) $\varrho_c \cong \varrho(M=1) \exp(-h)$, where $\varrho(M=1)$ is the density at the sonic point (typically 1 g cm^{-3}).

For the following example we choose $h = 10$. The assumption of a planar flow of the heated wall material is then well fulfilled and the density in the centre remains so low that the cavity may be considered as empty even for laser irradiation.

If it is noted that the sound velocity is related to the temperature by $s = (KT^\delta)^{1/2}$, (4) and (6), together with the expression $E = 4\pi R^2 S_s \tau$ for the energy delivered into the cavity, form a system of three equations for the six quantities E , S_s , h , τ , R , and T . If three of them are specified, the others are fully determined.

Let us assume that E (determined by the source), S_s (likely to be limited by the focussability of the source or through constraints arising from nonlinear effects during thermalization of the source energy), and h are given. In this case one finds for τ and R

$$\tau \cong 5.3 \times 10^{-3} E^{13/42} S_s^{-19/42} h^{-26/42},$$

$$R \cong 3.9 E^{29/84} S_s^{-23/84} h^{13/42}.$$

The temperature is then obtained from (4) by using the calculated value of τ , and N from (5).

If (as marked in Fig. 1) $E = 10^{12}$ erg, $S_s = 10^{14} \text{ W cm}^{-2}$, and $h = 10$, one obtains $\tau \cong 2 \times 10^{-9}$ s, $R = 0.2$ cm, $T \cong 5 \times 10^6$ K and $N \cong 40$. A black-body radiant energy flux of $\cong 4 \times 10^{15} \text{ W cm}^{-2}$ would thus be generated at $\cong 40$ re-emissions of the source energy.

If the numerical coefficients from the exact self-similar solutions are taken into account in the scaling laws, the situation is slightly less favourable;

for $(j, \mu') = (1, 1)$, N is reduced by a factor [4, 5] $\tilde{S}_p^{-24/21} = (1.33)^{-24/21} = 0.72$ to $N = 28$ for this example. It must be kept in mind, however, that our estimate represents in any case only an upper limit for the attainable conditions based on the maximum possible opacity. The extent to which N may be lowered because the real opacity of the wall material lags behind the maximum opacity remains to be seen from opacity calculations of high- Z materials including line transitions. It is merely noted that on the basis of free-free and free-bound opacities (but taking into account the numerical coefficient from the self-similar solution) one calculates for our example $N = 2.4$ as a lower bound for N [5].

Given the case that real opacities can come close to the maximal ones and that even higher pulse energies than 10^{12} erg may be realizable in the foreseeable future, our estimates suggest that temperatures of about 5×10^6 K and black-body radiant energy fluxes exceeding 10^{15} Wcm $^{-2}$ might be attainable in the laboratory. The major difficulty in realizing such high temperatures with laboratory sources and a chance of failure reside in the physics of source-matter interaction, which, at least in the laser case, may be rather complex at the high flux densities involved (this problem has been excluded in our analysis by the assumption of ideal coupling

between source and radiation). As long as it is not clear that such effects are entirely prohibitive, the possibility to transform pulsed power energy into incoherent, isotropic black-body radiation of short wavelength is felt to be of very great, if not decisive importance for the quantitative application of modern pulsed power sources. In so far as the basic independence of equilibrium radiation from the confining wall material and from the generation mechanism can be realized, the applications of these sources can be put on a universal basis. In particular, this scheme offers a potential solution for the fundamental difficulties arising from the requirement of uniform deposition of the source energy on an object in scientific and technical application. The coherence of laser light, for example, counteracts uniformity in direct deposition.

Acknowledgements

The authors would like to thank S. Witkowski for discussions and a critical reading of the manuscript.

This work was supported in part by the Commission of the European Communities in the framework of the Euratom/IPP Association.

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